PRESSURE

BEFORE STARTING ...

- Review :the calculation of the area of simple geometric figures
- Review: direct ratio and inverse proportion
- Review : “THE FORCES” and their measure
- Remember that since a body has a mass , it has a weight and that weight is a force
  $F_W = mg$

LEARNING OBJECTIVES

- The definition of pressure
- Pascal’s principle
- Stevin’s Law
- Archimedes’ principle
- Atmospheric pressure

UNDERSTANDING OBJECTIVES

- how to calculate the pressure of solids
- the relationship between pressure, area and force (quantities directly and inversely proportional)
- how pressure acts in fluids
- the importance of Pascal’s principle and its applications
- the importance of hydrostatic pressure and its consequences
- when an object floats or sinks in a liquid
- the importance of atmospheric pressure

APPLICATION OBJECTIVES:

- to solve simple problems

LESSON PLAN

1. **What is pressure (brainstorming)**
2. **How to calculate pressure mathematically**
3. **SI unit measure of pressure**
4. **Simple calculations of pressure in solids**
5. **Pressure and fluids**
   - **Pascal principle and its consequences**
   - **Stevin’s Law and its consequences**
   - **Archimedes’ principle and the buoyant force (the flotation of bodies)**
6. **Atmospheric pressure and its measure**

1. WHAT IS PRESSURE?

BRAINSTORMING

There are many physical situations where pressure is the most important variable.

Some people think of pressure as a push or a force, but this is not correct, it is something more definite than that.

Think about some ordinary situations (as described below) and try to answer to the following questions:

(ATTENTION! -*You're not allowed to just answer "No"!)*

QUESTIONS

1. Why can you sit on a chair but if an elephant does the same the chair breaks down?
2. Have you ever wondered why camels or elephants have large feet?
3. Why don’t skiers sink into the soft snow while this happens if you walk on soft snow with shoes?
4. Why has a bulldozer wide caterpillar tracks and not wheels?
5. Imagine you have to drive a stake into the ground. You have hit with a heavy hammer but with little success. Is there something else you can do to make your job easier?
6. Why is it better for you to use a sharp knife when you are peeling an apple?
7. Why is it better to have a sharp needle than a dull one if you must get an injection?

ANSWERS

1. This happens because the elephant has a bigger mass than you or, more exactly, his weight is bigger than yours (remember! Weight is a force)
2. Camels and elephants need to be able to walk around on sand or on ground without sinking into it. This means that they need to reduce their **pressure** on the ground. Large feet mean a large area of contact, and thus less pressure.
3. For the same reason skiers don’t sink into the ground: skies have a large surface, they keep you from sinking into the snow by reducing the pressure you put on the snow with each step. You don’t weigh less when you put skies or snow shoes, but the force of your **weight** is spread over a much wider area so that there is **less pressure.**
4. If the bulldozer had four wheels like a car , it would exert a much larger pressure and could sink into the ground.
5. You could sharpen the end to a point. It is much easier to drive a pointed stake into the ground because greater pressure can be exerted. All the force is concentrated on a very small area and so you are concentrating the force on that point.
6. Because if the knife is sharp, then the area of contact is small and you can peel with less force exerted on the blade.
7. Because the smaller area of contact implies that less force is required to push the needle through the skin.

2. PRESSURE UNITS AND CALCULATIONS

Now we are able to understand that pressure depends on two quantities:

- the **Force** (in Newtons)
- the **Area** it's pressing on (in square metres)

■ **PRESSURE IS THE FORCE ACTING PERPENDICULARLY ON THE UNIT AREA.**

This means the number of newtons pressing on each square metre of surface.

$$\text{PRESSURE} = \frac{\text{FORCE}}{\text{AREA}}$$

Mathematically;:

$$p = \frac{F}{A}$$

where:
p is the pressure
F is the **normal** force
A is the area

■ We can also say that " **PRESSURE IS THE RATIO BETWEEN THE FORCE ACTING PERPENDICULARLY TO A SURFACE AND THE AREA OF THE SURFACE ITSELF"**

Pressure describes the concentration of a force. So you can think of pressure as being a measure of ***how a force is concentrated.***

We can see that **pressure is directly proportional to the force and inversely proportional to the surface.**

It's always easier to cut food with a sharp knife than with a blunt one. A sharp knife is one that has a very thin edge to the blade, allowing the force applied to be concentrated on a **small area**, so that the pressure you can apply is greater. The opposite is also true.

- **Pressure increases when the force applied on the same surface increases**
- **Pressure is reduced when a force is spread over a large area**
- **Pressure increases when the force is spread over a small area**

Is pressure a scalar or a vector quantity?

We know that force is a vector while area is a scalar. But since we consider only the **normal** component of the force, then **pressure is a scalar.**

Since force is expressed in newtons, and area in square metres, then **the SI unit of pressure is the newton per square metre, N/m^2.**

A force of 1 newton acting on a surface of 1 square metre is called pascal (**Pa** for short,**)** named after the French scientist Blaise Pascal (1623-1662). (Pressure can be also be expressed in N/cm^2)

$$1\ \text{Pa} = \frac{1\ \text{N}}{1\ \text{m}^2}$$

One pascal (1 Pa) is a very small pressure, and meteorologists use hectopascals (hector = 10^2 pascals) when expressing atmospheric pressure.

For an object sitting on a surface, the force pressing on the surface is the weight of the object, so that if you have the mass of the object you can calculate its weight **W = m· g**

- Since pressure is the force per unit area, we also can say that **a force is created by the pressure exerted on an area:**

FORCE = PRESSURE x AREA

$$F = p \cdot A$$

and if we want to calculate the area of the surface over which the pressure is acting, then:

AREA = FORCE / PRESSURE

$$A = \frac{F}{p}$$

SIMPLE CALCULATIONS

1) EXAMPLE SOLVED
Three men have the following masses:
m_1 = 40 kg
m_2 = 75 kg
m_3 = 110 kg

They sit on a chair. If the area of one leg of the chair is 9.0 cm^2 , calculate the pressure that each of them exerts on the ground.

SOLUTION
Calculate the weight of the three man: W = mg
W_1 = 40 kg · 9.81 N/kg = 392 N
W_2 = 75 kg · 9.81 N/kg = 738 N
W_3 = 1110 kg · 9.81 N/kg = 1 079 N

Calculate the total area of the legs of the chair:
A = 9 cm^2 · 4 = 36 cm^2 = 36 · 10^{-4} m^2

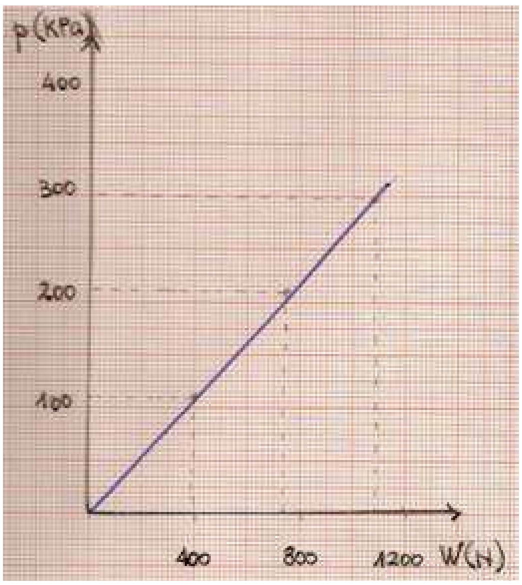
Now we can calculate the pressure $p = \frac{W}{A}$ in each case:

$$p_1 = \frac{392\ N}{36 \cdot 10^{-4}\ m^2} = 1.1 \cdot 10^5\ Pa$$

$$p_2 = \frac{736\ N}{36 \cdot 10^{-4}\ m^2} = 2.1 \cdot 10^5\ Pa$$

$$p_3 = \frac{1079\ N}{36 \cdot 10^{-4}\ m^2} = 3.0 \cdot 10^5\ Pa$$

Now plot the graph.



You can see that **PRESSURE IS DIRECTLY PROPORTIONAL TO THE FORCE:**

$$\frac{p}{F} = k$$

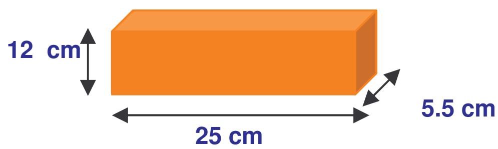
where the constant of proportionality is the inverse of the area:

$$\frac{p}{F} = \frac{1}{A}$$

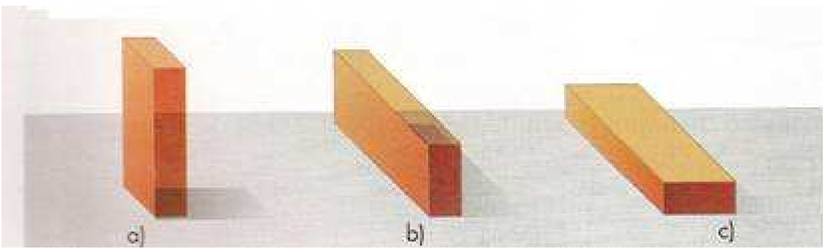
in the example, considering the first case:

$$\frac{1.1 \cdot 10^5\ N/m^2}{392\ \ N} = 281\ m^{-2} = \frac{1}{0,0036\ m^2}$$

2) EXAMPLE SOLVED
A brick with a mass m = 3.0 kg has the following dimensions:
length = 25 cm
width = 12 cm
height = 5.5 cm



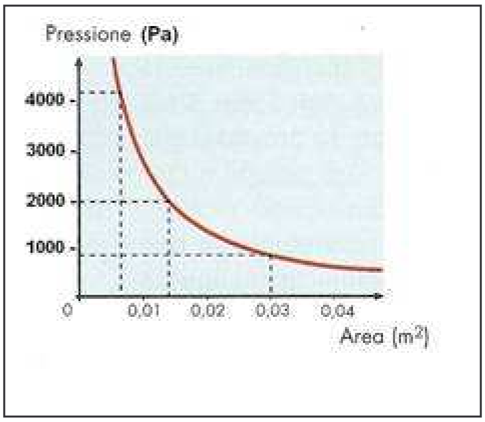
Calculate the pressure exerted by the brick in each of its positions.



SOLUTION

Weight is a force:
W = mg = 9,81 N/kg · 3.0 kg = 29 N

and this force is acting (pressing on) on the base of the block, whose areas are:



a) $A_1 = 5.5\ cm \times 12\ cm = 66\ cm^2 = 66 \cdot 10^{-4}\ m^2$

$$P_1 = \frac{29\ N}{66 \cdot 10^{-4}\ m^2} = 4400\ Pa$$

b) $A_2 = 25\ cm \times 5.5\ cm = 137.5\ cm^2 = 137.5 \cdot 10^{-4}\ m^2$

$$P_2 = \frac{29\ N}{137.5 \cdot 10^{-4}\ m^2} = 2100\ Pa$$

c) $A_3 = 25\ cm \times 12\ cm = 300\ cm^2 = 300 \cdot 10^{-4}\ m^2$

$$P_3 = \frac{29\ N}{300 \cdot 10^{42}\ m^2} = 970\ Pa$$

We can see that:

1. a brick **in different orientations has a different area in contact with the surface** and therefore exerts a **different pressure**;
2. since the weight is always the same **pressure is inversely proportional to the area of the surface** and **the constant is the weight of the brick**.

CHECK YOUR WORK

1) An office safe has a weight of 500 N. If the area of the base is 1.25 square metres, what is the pressure on the floor of the office?

2) A physics teacher has a mass of 75 kg. If his feet have an area of 0.025 square metres, what pressure does he exert on the ground? (Remember he has two feet!)

3) A marble statue rests on a circular surface with a radius of 50 cm and it exerts a pressure of 6 190 Pa. Calculate the mass of the statue.

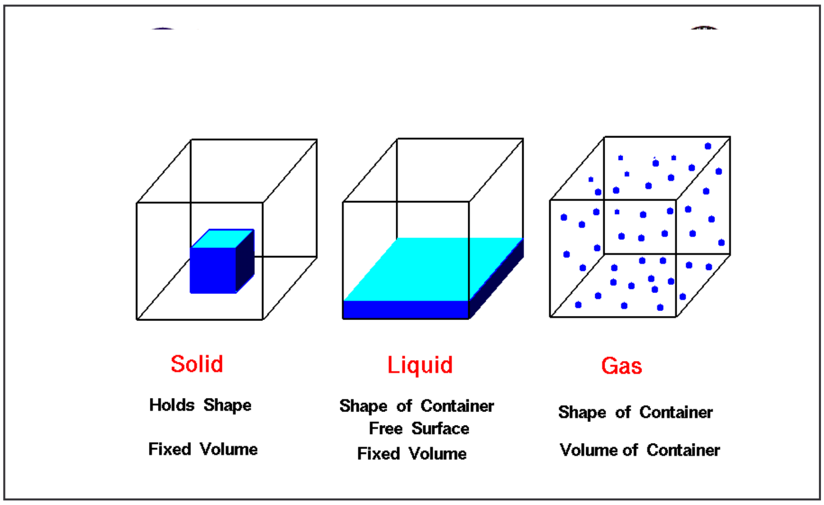
3. PRESSURE AND FLUIDS: PASCAL'S PRINCIPLE

First of all we must answer the following questions:

1. **"What's a fluid?"**
2. **Why do fluids exert a pressure?**

- We know that the states of matter are solid, liquid, gas an plasma. Fluids are a subset of the states of matter and include **liquids, gases and plasma.** The term fluid is often used as being synonymous with "liquid". This can be erroneous and sometimes clearly inappropriate because the term liquid doesn't involve gaseous state.

  Fluids have common properties that are distinct from solids. Fluids don't have a specific shape as solids do. Instead, fluids take the shape of their containers.



A fluid is defined as a substance that continually deforms (flows) under an applied shear stress or simply, **a fluid is any material capable of flowing All liquid and gases are fluids.**

- **Fluids**, as solids do, **exert a pressure because of their weight**. A solid object presses downwards on its base; in a **fluid**, pressure acts not only from the top to the bottom but it is transmitted undiminished to all directions including upwards.

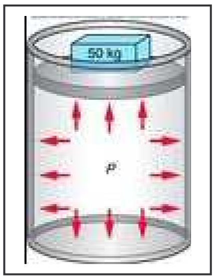
PASCAL'S PRINCIPLE

Pascal's Principle states that:

"if you apply a pressure to a fluid that is *confined* (or *can't flow to anywhere*), **the fluids will then *transmit* that same pressure in all directions at the *same rate***"

or :

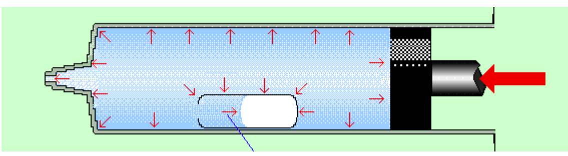
"pressure applied to a completely enclosed fluid is transmitted without loss to all the parts of the fluid and to the walls of the container"



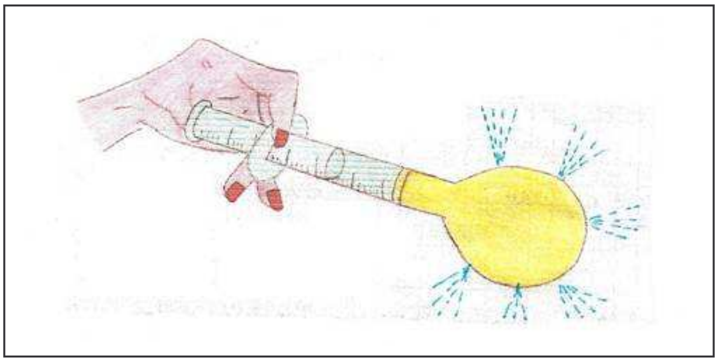
EXPERIMENTS

It's useful to carry out simple experimental activities to promote in the classroom observations and discussion on the behaviour of liquids.

EXPERIMENT 3.1. – For injecting the liquid of a siring the force is communicated to the liquid by the piston pressing on the liquid that flows in the graduated cylinder.

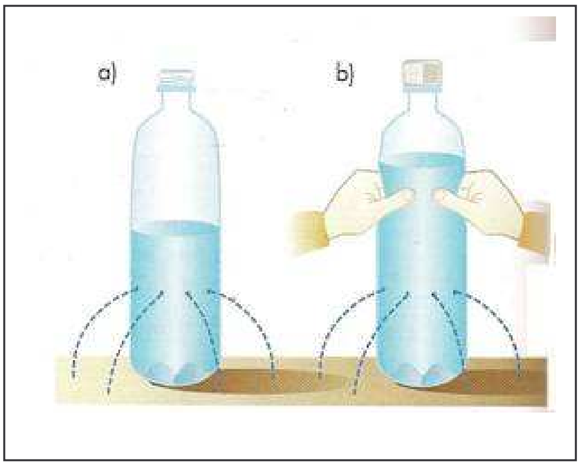


EXPERIMENT 3.2 – Take a siring and a balloon. Fill the siring with water. Make little holes all around the balloon. Insert the balloon on the top of the siring and press the piston.



In accordance with Pascal's principle water sprays from the holes in all directions and independently of the direction of the pressure exerted by the piston.

EXPERIMENT 3.3 – Punch few holes in a small plastic bottle, fill it with water, and the press it: water will spurt out in all directions.

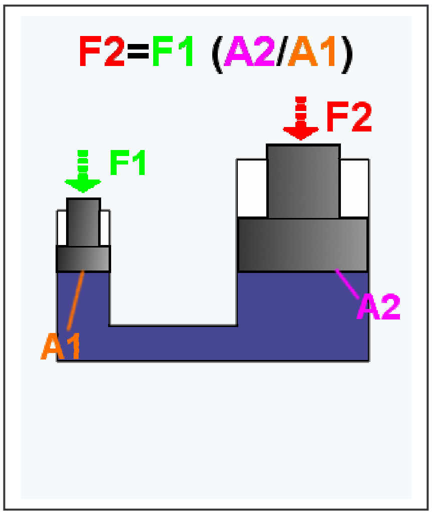


Pressure acts in all directions

APPLICATIONS OF PASCAL'S PRINCIPLE

These ideas benefit us every day.

Many technical equipment relate to Pascal's principle: the most important is a system called **hydraulic press.** A hydraulic press is **a hydraulic mechanism for applying a large lifting or compressive force.** Hydraulic systems use a **incompressible fluid**, such as **oil** or **water**, **to transmit forces from one location to another within the fluid.**



At one end of the system there is a piston with a small cross-sectional area driven by a lever to increase the force. Small-diameter tubing, filled with mineral oil, leads to the other end of the system which has a bigger crossing sectional-area.

The figure displays two joined cylindrical chambers. They both have different diameters and include a connected tube; they are filled with the same liquid. A force F_1 is applied to the small piston of area A_1. This force exerts a pressure:

$$P = \frac{F_1}{A_1}$$

This pressure is transmitted through a fluid (a liquid or an oil) to a larger piston of area A_2.

Since the first cylinder is connected to a second cylinder of a larger diameter, **the pressure inside that cylinder is the same P** and this pressure will be equal to:

$$P = \frac{F_2}{A_2}$$

Since the pressure is the same on both sides, then:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

or:

$$F_2 = F_1 \frac{A_2}{A_1}$$

Since the second piston is larger, it produces a larger force than the force applied to the smaller piston: for example if the area of the first piston is 0.015 m^2 and the area of the second piston is 0.45 m^2 and on the first piston is applied a force of 130 N then on the second piston is produced a larger force:

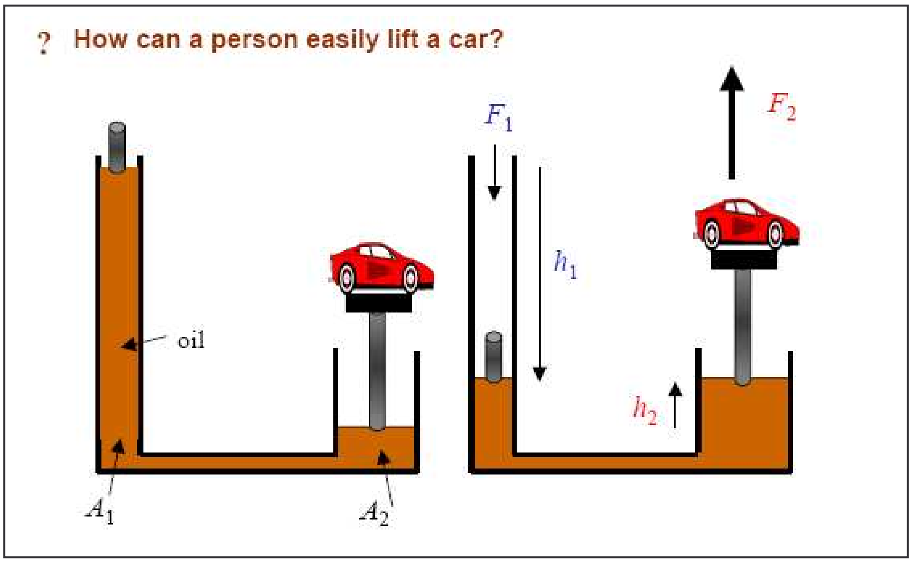
$$F_2 = F_1 \frac{A_2}{A_1} = 131N \frac{0.45 m^2}{0.015 m^2} = 3\ 930N$$

HYDRAULIC CAR LIFT, HYDRAULIC BRAKES, HYDRAULIC JACKS, FORKLIFTS and LIFT TRUCK all make use of this principle

1. Hydraulic Car Lift

The hydraulic car lift relates to Pascal’s Principle. It’s used in garage’s to transport a car off the ground to repairing.

A hydraulic lift for automobiles is an example of a force multiplied by hydraulic press, based on Pascal’s principle. The fluid in the small cylinder must be moved much further than the distance the car is lifted.



EXPLANATION

1. Pressure is exerted on the fluid in the small cylinder, usually by a compressor.
2. Pressure is transmitted equally to the all parts of an enclosed static fluid according with Pascal's law.
3. Since the pressure is the same and it is exerted over a much larger area, it gives a multiplication of the force that lifts the car.

1) EXAMPLE SOLVED

A hydraulic car lift has a pump piston with radius r_1 = 0.0120 m. The resultant piston has a radius r_2 = 0.150 m. What force must we exert on the first piston to lift a 6 000 N car?

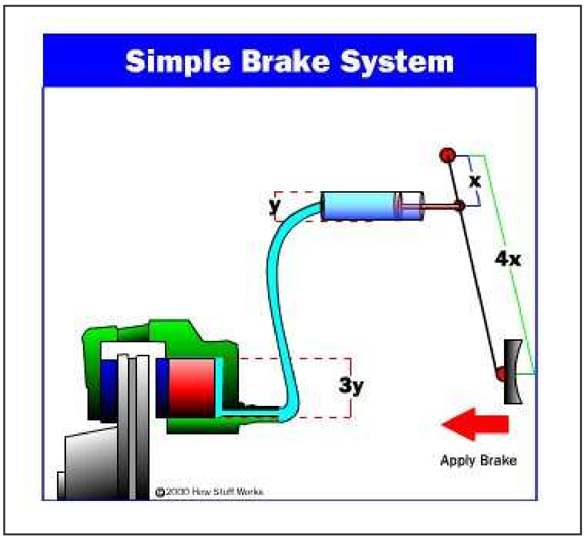
SOLUTION

$A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2$

Then the ratio of the areas is:

$$\frac{A_2}{A_1} = \frac{(0.15m)^2}{(0.012m)^2} = \frac{225 \cdot 10^{-2}m^2}{144 \cdot 10^{-4}m^2} = 156$$

you would have to exert only 6000 N/156 = 38 N on the fluid in the small cylinder to lift the car.

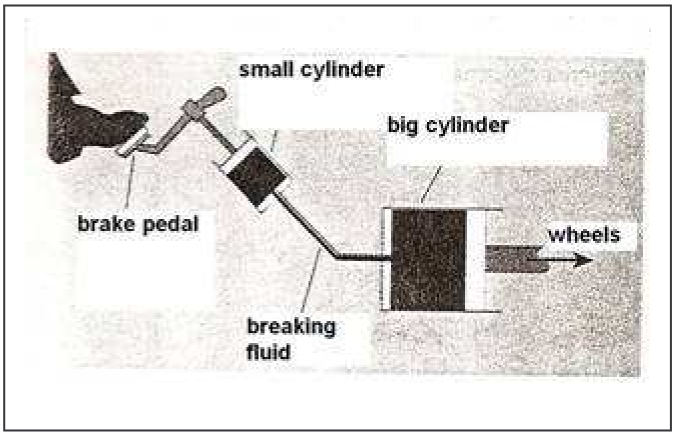


2. Hydraulic Car brakes

When your father or mother brakes the car, the brake pedal makes the piston to act and this applies a force on the master cylinder (the second cylinder that is larger than the first one). The main pipe is connected to four slave pistons that activate the brake pads of your car to the brake rotor, which in turn makes the car come to a stop.

2) EXAMPLE SOLVED

The braking gear of a car is composed of a small cylinder with an area $A_1 = 4{\cdot}10^{-4}\ m^2$ and a bigger one with an area $A_2 = 2.4{\cdot}10^{-3}\ m^2$ If your foot exerts a force of 90 N, calculate the force applied on the bigger cylinder.



SOLUTION

DATA

$A_1 = 4{\cdot}10^{-4}\ m^2$
$A_2 = 2.4{\cdot}10^{-3}\ m^2$
$F_1 = 90$ N
For the Pascal's principle:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Since :

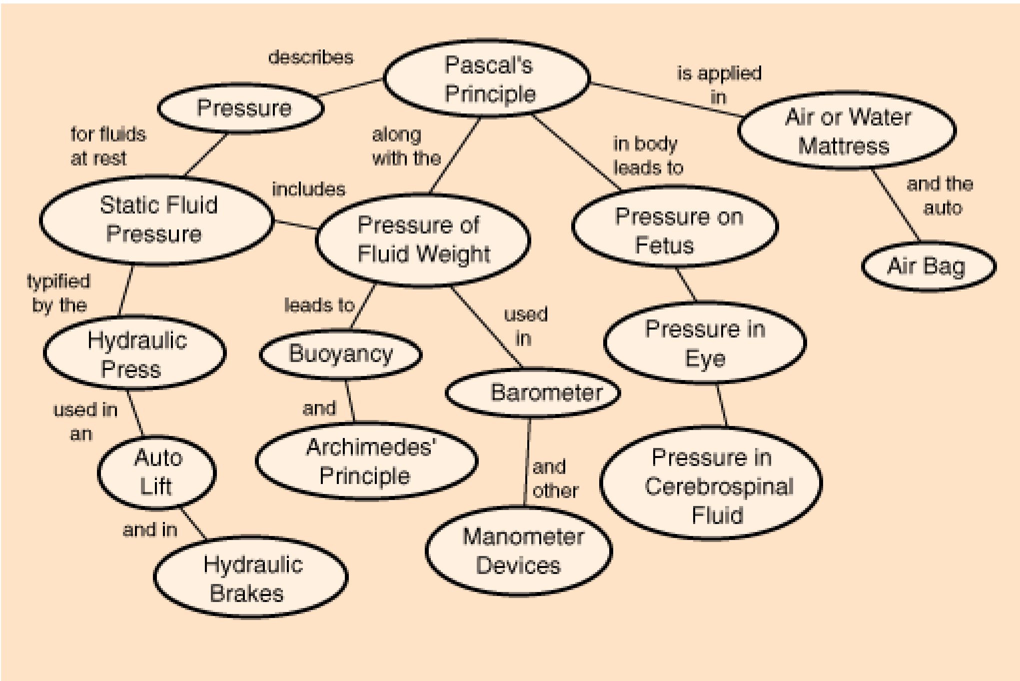
$$\frac{A_2}{A_1} = \frac{2.4 \cdot 10^{-3}\ m^2}{4 \cdot 10^{-4}\ m^2} = 6$$

The force exerted on the second cylinder is:

$$F_2 = 90N \times 6 = 540\ N$$

As you can see, your foot exerts a small force on the pedal but the pressure is transmitted through the braking fluid to the bigger cylinder that has a larger area and this exerts a larger force that stops the car.

Pascal's principle has many other applications (see below):



4. STEVIN'S LAW: STATIC FLUID PRESSURE

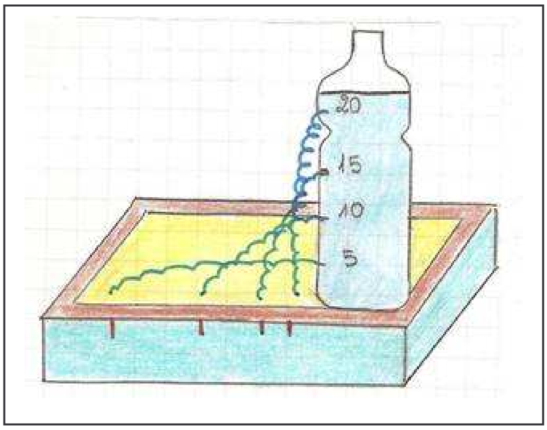
Think about the following situations.
When you dive under water, you have the weight of the water above you, pressing on your body. If you have ever dived to the bottom of a swimming pool , you will have felt the pressure on you eardrums. This is because the air inside you ears is easily squeezed but the solid and liquid parts of your body can withstand the pressure.

Why does water exert a pressure? The pressure in a static fluid (water) arises from the weight of the fluid .
The pressure exerted by a liquid is called HYDROSTATIC PRESSURE.

How does pressure in a liquid depend on depth?

EXSPERIMENT 4.1– To investigate how pressure depends on depth.

Take a plastic bottle and make three holes at different heights. Fill the bottle with water: the water jet from the lowest hole travels furthest while the jet from the top hole only travels a short distance.



CONCLUSION: pressure increases with depth. The pressure is caused by the weight of the liquid above each particular hole. The greater the depth, the greater is the pressure.

STEVIN'S LAW (named after Simon Stevin (1548-1620) a famous Dutch scientist) states that:

"The pressure exerted by a static fluid depends only upon the depth of the fluid, the density of the fluid, and the acceleration of gravity."

$$p_{hyd} = d \cdot g \cdot h$$

where:

p_{hyd} is **the hydrostatic pressure**
d is the **density of the liquid**
g is **the acceleration of gravity**

In fact :

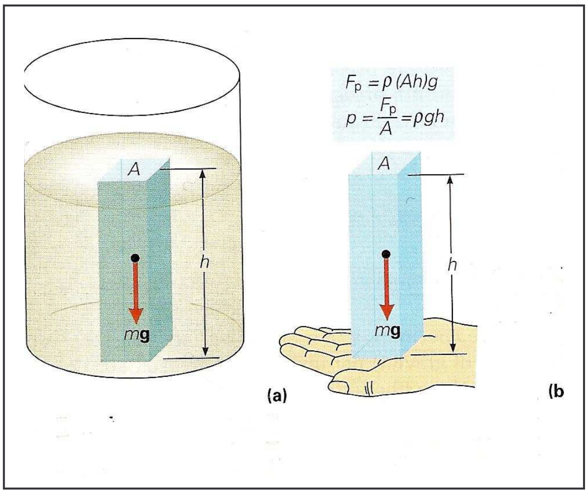
$p = \frac{P}{A}$ where P is the weight of the liquid

$P = m \cdot g$ and $m = d \cdot V$ where d= density

$p = \frac{d \cdot V \cdot g}{A}$ but $\frac{V}{A} = h$

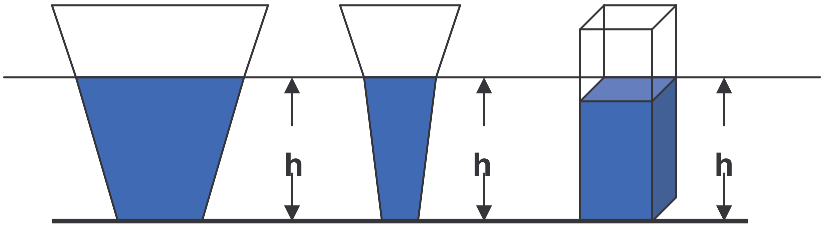
so that:

$p = d \cdot g \cdot h$



The most remarkable thing about this expression is what it does **not** include. The fluid pressure at a given depth **does not depend upon the total mass or total volume of the liquid**.

Consider three vessels with a different shape filled with water. What pressure is exerted by water on the bottom of each vessel?



Since static fluid pressure **doesn't** depend on the shape, total mass or surface area of the liquid, but it depends only on **the height of the column water, the pressure in all the vessels shown above is the same, independently of the shape of the container because the liquid has always the same height.**

EXAMPLE SOLVED

What is the pressure exerted by sea water on a sub at the depth of 10 m?

ANSWER

Since the density of the sea water is about d = 1020 kg/m^3, then:

$$p_{hyd} = 1020\ kg/m^3 \cdot 9.81\ N/kg \cdot 10\ m = 100\ 062\ Pa$$

This is a great pressure if we compare this value with the pressure exerted by a man with a mass of 75 kg on an area of 400 cm^2 (the total area of his feet).

Since 101 326 Pa = 1 atm we can say that:

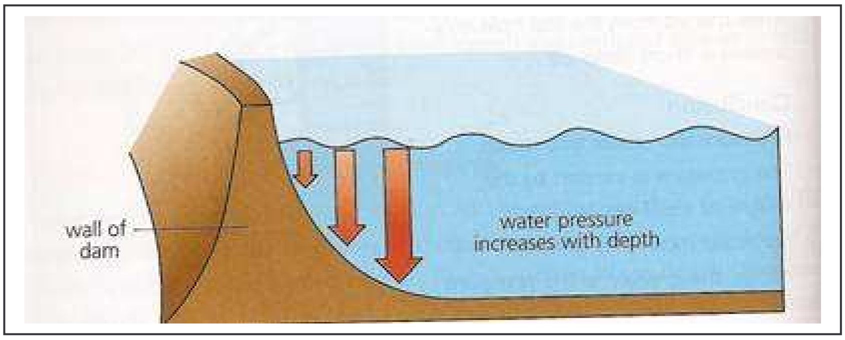
" For each 10 m of depth , the pressure increases by the equivalent of 1 atmosphere"

CONSEQUENCES OF STEVIN'S LAW

Deep-sea divers have suits made of strong thick materials that can withstand the very high pressures in deep water.

How does that affect scuba divers as they go up and down in the sea? The human body is made mainly of solids and liquids, and these are strong enough to withstand strong pressure. However, blood carries dissolved gases. When a diver is a few hundred metres below the surface, the air that he or she breaths is pressurised to push it into his or her lungs. From there it goes into the blood and dissolves. Problems can developed if the diver try to return to the surface quickly. As the diver comes up, the water pressure on his or her body decreases. This causes the air in the blood to form tiny bubbles, a condition best known as " the bends". This can seriously damage the blood vessels in the brain and other organs. Divers therefore have to ascend from great depths slowly or in stages.

Stevin's law explains why **dams must be built with thicker walls at the bottom.**



One of the first principles that the engineers who design dams have to bear in mind is **the deeper you go in a liquid, the greater the pressure becomes.** This means that the pressure on the bottom of the dam is much grater than at the top, and so the wall of the dam must be thicker at the bottom.

5. ARCHIMEDES' PRINCIPLE AND BUOYANT FORCE

Buoyancy is the ability of an object to float or **the power of a liquid to make an object float.**

In physics **buoyancy** is **the upward force on an object produced by the surrounding fluid** (a liquid or a gas) **in which it is fully or partially immersed.**

Think about the following evidences:

a. it's easier to swim in the sea than in a swimming pool

b. if you try to rise up a person in the sea water he seems lighter

c. a wood or a ship floats on the water

d. a balloon floats in the air

On what do these phenomena depend? They are due to the **Archimedes' principle.**

Archimedes' principle states that:

" a body wholly or partly immersed in a fluid is buoyed up by a force equal to the weight of the displaced fluid"

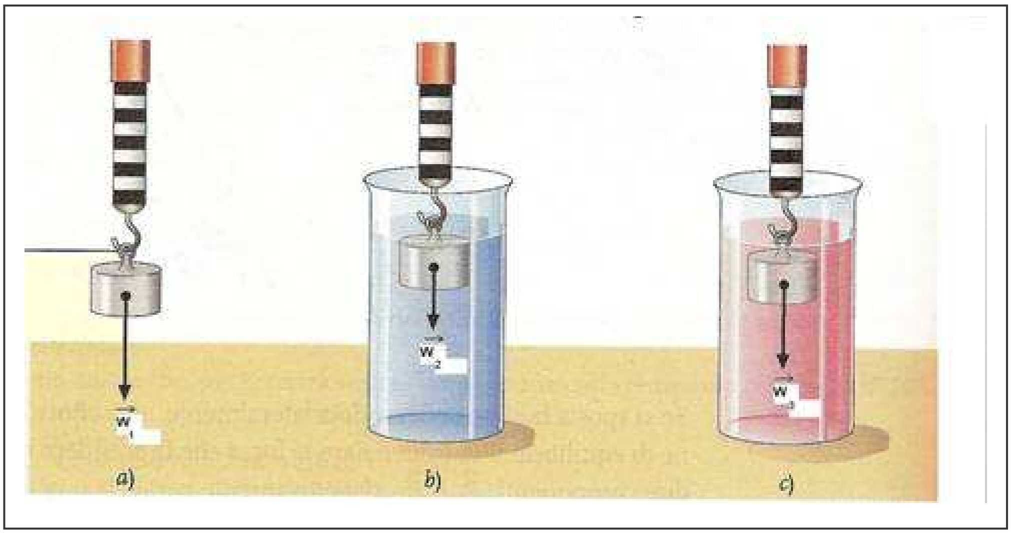
In other words, when a body is partially or completely immersed in a liquid, then it experiences an upward buoyant force which is equal to the magnitude of the weight of the liquid displaced by the immersed part (or volume) of the body.

This force enables the object to float or at least to seem lighter.

Buoyancy is important for many vehicles such as boats, ships, balloons, and airships, and plays a role in diverse natural phenomena such as sedimentation.

EXPERIMENT 5.1

1. **Tie up a stone with a string and hang it to a dynamometer**
2. **Write down its weight W_1**
3. **Immerse the stone in water and write down its new weight W_2**
4. **Repeat the experiment using alcohol instead of water and write down its weight W_3**



You can notice that $W_2 < W_3 < W_1$

We must suppose that **water (or alcohol but in less measure) exerts a force F_B directed upward on the body immersed that causes the apparent loss of weight.**

We can call W_2 (or W_3) **W_A, apparent weight.**

The difference between W an $W_{A,}$ is called **buoyant force F_B**, that is:

$$W - W_A = F_B$$

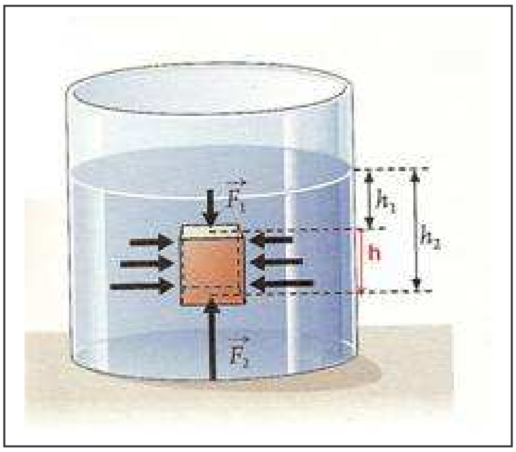
where:

W is the true (vacuum) weight of the object
W is the apparent weight of the object

The buoyancy acting on the object reduces the weight of the object. The buoyancy acting on the body, when it is fully immersed in the water, is equal to the weight of the water displaced by the object.

DEMOSTRATION

We know (Stevin's law) that pressure increases with depth below the surface of a liquid. Any object will see different pressures on its top and bottom, with the pressure on the bottom being higher. This difference on pressure causes the buoyancy force.



Using a cube as an example, the pressure on the top surface is:

$$p_1 = dgh_1 \quad \text{(d is the density of the liquid)}$$

and the pressure on the bottom is:

$$p_2 = dgh_2$$

With $p_2 > p_1$ since $h_2 > h_1$.

So, the acting force on the top is :

$$F_1 = p_1A = dgh_1A$$

and the force acting on the bottom is:

$$F_2 = p_2A = dgh_2A$$

with $F_2 > F_1$.

The resultant force is a upward force that is the **buoyancy force F_B**:

$$F_B = F_2 - F_1 = dgh_2A - dgh_1A = dgA\,(h_2 - h_1)$$

Since $(h_2 - h_1)$ is the length of the edge of the cube h then $A(h_2 - h_1)$ **is the volume of the immersed cube and this is equal to the volume of the displaced fluid.** So

$$\mathbf{F_B = dgAh = dgV_i}$$

where:
F_B is the buoyancy force
d is the density of the fluid
V_i is the submerged volume of the body or, that is the same, the displaced volume of the fluid

$V_i \equiv V_{\text{displaced fluid}}$

by multiplying on right and left for d, the density of the fluid,

$dV_i = dV_{\text{displaced fluid}}$

and since

$dV_{displaced\ fluid} = m_{fluid}$

then:

$dV_i = m_{displaced\ fluid}$

by multiplying for g (the gravity acceleration):

$gdV_i = m_{displaced\ fluid}\ g$

but

mg=W

so:

$$F_B = gdV_i = m_f g = W_f$$

where W_f **is the weight of the displaced fluid.**

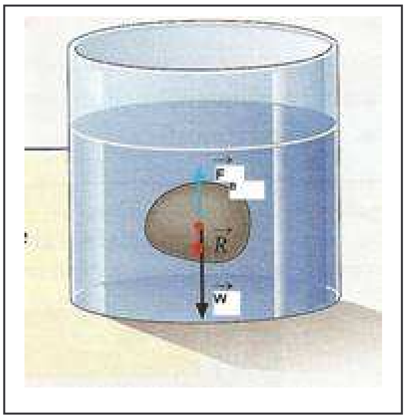
We can see that **the buoyancy of an object depends,** therefore**, on only two factors: the submerged volume of the object and the surrounding density of the fluid.** The greater the volume of the object and the surrounding density of the fluid are, the more buoyant force it experiences. So we can understand why the buoyant force is less in alcohol than in water: because the density of alcohol is less than density of water.

The total force on the object is thus the net force of buoyancy and the object's weight:

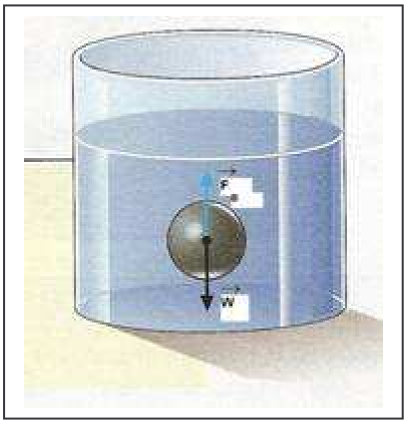
$$F_{net} = mg - dVg$$

If the buoyancy of an object exceeds its weight, it tends to rise. An object whose weight exceeds its buoyancy tends to sink.

Now we can understand why some objects can float on water and others tend to sink.

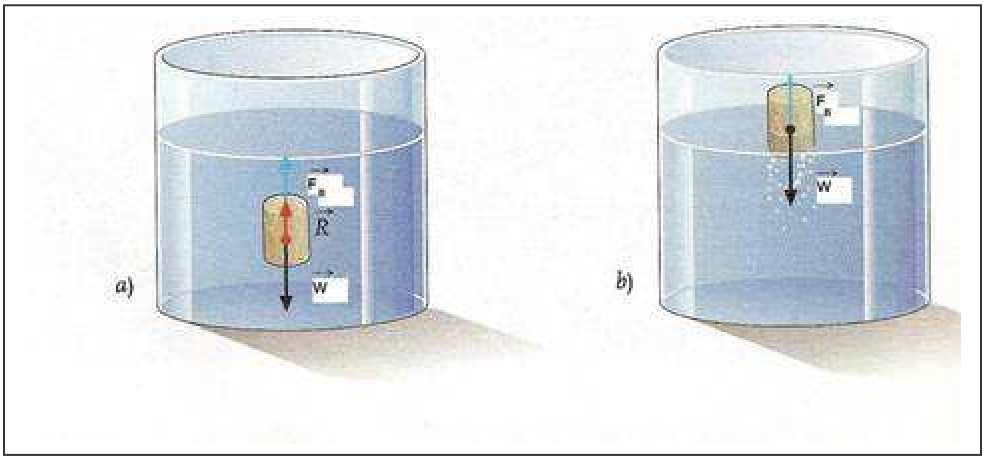


1. If $W > F_B$ or $d_o > d_f$ that is **the weight of the object exceeds the buoyancy** or in other words **the density of the object is greater than the density of the fluid**, the object **will sink.**

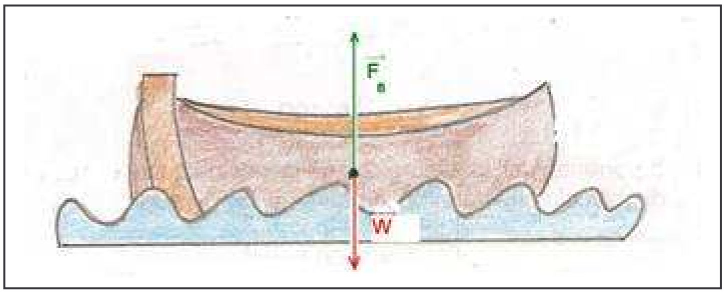


2. If **W = F_B** or **$d_o = d_f$** that is **the weight of the object equals its buoyancy** or in other words **the object has exactly the same density as the fluid**, it **will tend neither to sink nor to float.**

3. If **W <F_B** or **$d_o < d_f$** that is **the buoyancy of the object exceeds its weight** or in other words **the density of the object is less than the density of the fluid**, the object **will float on the fluid with a part of its volume rising out the liquid.**



How is it possible **to calculate the immersed volume of an object**? Consider a boat:



There are two forces acting on the boat: its weight W and the buoyant force F_B that have opposite directions. At the equilibrium:

$$W = F_B$$

If:

V_i is the immersed volume of the boat and
d_f is the density of the liquid

then:

$$mg = d_f g V_i$$

if d_o is the density of the object (the boat in this case) and V is its total volume:

$$d_c V \not{g} = d_f \not{g} V_i$$

so:

$$d_o V = d_f V_i$$

and then::

$$\frac{V_i}{V} = \frac{d_o}{d_f}$$

that is:

$$\%V_i = \frac{d_o}{d_f} \cdot 100$$

For a body that floats on a liquid **the percentage of the volume immersed is equal to the ratio between the density of the object and the density of the liquid in which it is immersed**.

WORKED EXAMPLE – Calculate the percent volume of a cork ($d_{sughero}$ = 240 kg/m^3):
a) in water (d = 1 000 kg/m^3)
b) in mercury (d = 13 600 kg/m^3)

SOLUTION

a) $\% V_i = \frac{d_{cork}}{d_{water}} \cdot 100 = \frac{240 kg/m^3}{1000 kg/m^3} \cdot 100 = 24\%$

b) $\% V_i = \frac{d_{cork}}{d_{mercury}} \cdot 100 = \frac{240 kg/m^3}{13\ 600 kg/m^3} \cdot 100 = 1.7\%$

SHIPS AND SUBMARINES

Why does a ship or a submarine float ?

A ship floats because although it is made of steel, which is more dense than water, it encloses a volume of air and the resulting shape has a less average density than water.

Submarines rise and dive by filling large tanks with seawater. To dive, the tanks are opened to allow air to exhaust out the top of the tanks , while the water flows in from the bottom. Once the weight has been balanced so the overall density of the submarine is equal to the water around it, it has a neutral buoyancy and will remain at that depth. To re-emerge the water in the tanks is pumped out.

THE TIP OF THE ICEBERG

We know that if the density of an object is less than the density of the water, the percentage of its immersed volume is:

$$\%V_i = \frac{d_c}{d_f} \cdot 100$$

You know that the icebergs are very dangerous for the ships that sail in the Northern seas. Why?

Since:

$$d_{ice} = 900 \text{ kg/m}^3$$
$$d_{water} = 1\ 000 \text{ kg/m}^3$$

so:

$$V_{i\ iceberg} = \frac{900}{1000} \cdot 100 = 90\%$$

What does this mean? It means that an iceberg floats on the water with only 1/10 (one tenth) of its volume rising out the water and that the 90% (ninety percent) of its total volume is under the water! For example, if a ship meets with an iceberg whose tip is 10 m height, under the water there is an ice floe 100 m height! And this is very dangerous.

APPLICATIONS OF ARCHIMEDES’ PRINCIPLE

THE DENSITY OF A BODY

A man weight 720 N. He is secured with a sling and then immersed in a water tank till his chin. The sling is suspended from a balance that measures his apparent weight. He breaths out all the air he can and put his head under the water, so his body is completely immersed. If his apparent weight is 34,3 N calculate:

- **a. his volume**
- **b. his density**

SOLUTION

$W = 720$ N
$W_A = 34,3$ N (is the apparent weight)

$W - W_A$ is the buoyant force

$W - W_A = F_B = dg\, V_i$

$720 \text{ N} - 34,3 \text{ N} = 1000 \text{ kg/m}^3 \cdot 9,81 \text{ N/kg} \cdot V_i$
$686 \text{ N} = 9810 \text{ N/m}^3 \cdot V_i$

$$V_i = \frac{686}{9810} \text{ m}^3 = 6,99 \cdot 10^{-2}\, m^3$$

$$m = \frac{W}{g} = \frac{720N}{9,81\ N/kg} = 73,4 \text{ kg}$$

$$d = \frac{m}{V} = \frac{73,4 \text{ kg}}{6,99 \cdot 10^{-2}\, m^3} = 1050 \text{ kg/m}^3$$

EXAMPLE SOLVED – A piece of wood (d = 655 kg/m³) has the shape of a cube, with an edge of 15.0 cm.
What is the mass of the displaced water (d = 1 000 kg/m³) by the piece of wood when it floats?

SOLUTION

1. Calculate the volume of the piece of wood $V = l^3 = (15.0 \cdot 10^{-2}\ m)^3 = 3{,}38 \cdot 10^{-3}\ m^3$
2. Calculate the percentage of the immersed volume $\%V_i = \frac{d_o}{d_f} 100 = \frac{655\ kg/m^3}{1\ 000\ kg/m^3} \cdot 100 = 65{,}5\%$
1. Calculate the immersed volume $V_i = 3{,}38 \cdot 10^{-3}\ m^3 \cdot \frac{65{,}5}{100} = 2{,}21 \cdot 10^{-3}\ m^3$

According to Archimedes' principle the buoyant force is:

$$F_B = dgV_i = 1\ 000\ kg/m^3 \cdot 9{,}81\ N/kg \cdot 2{,}21 \cdot 10^{-3}\ m^3 = 21{,}7\ N$$

and this force is equal to the weight of the displaced water:

$W_{water} = 21{,}7\ N$ so that $m_{water} = \frac{21{,}7\ N}{9{,}81\ N/kg} = 2{,}21\ kg$ that is $2{,}21 \cdot 10^{-3}\ m^3$

6. THE ATMOSPHERE: AIR

The **Earth's atmosphere** is a layer of gases surrounding the planet Earth and retained by the Earth's gravity. It contains roughly (by molar content/volume) 78.08% nitrogen, 20.95% oxygen, 0.93% argon, 0.038% carbon dioxide, trace amounts of other gases, and a variable amount (average around 1%) of water vapor. This mixture of gases is commonly known as **AIR**.

PROVING THAT AIR EXISTS

Air is invisible, so how can you know that it exists? As it is composed of gases it is made of matter and like other forms of matter, it has a mass and occupies a space.

EXPERIMENT 6.1 – To show that air has mass

1. **Find the mass of an empty balloon.**
2. **Fill the balloon with air, tie it and again measure its mass.**
3. **The mass will have increased because of the air in the balloon.**

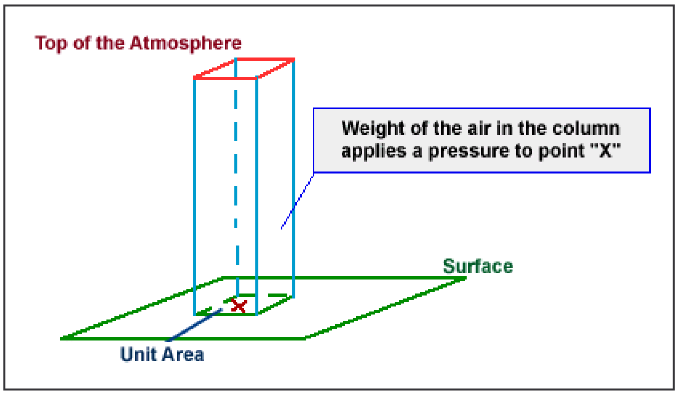
EXPERIMENT 6.1 – To show that air occupies space

Blow up a balloon: the more air that is blown into it, the bigger it becomes

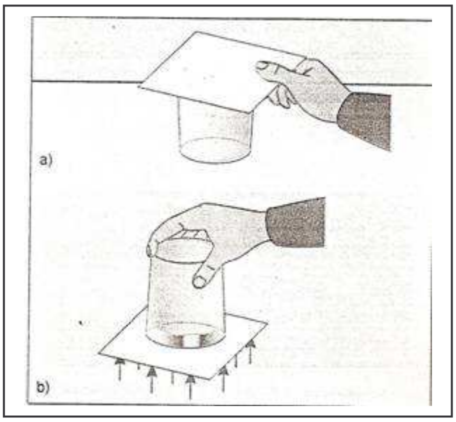
6.1- ATMOSPHERIC PRESSURE

Atmospheric pressure is the pressure at any point in the Earth's atmosphere. In most circumstances atmospheric pressure is closely approximated by the **hydrostatic pressure** (remember Stevin's law; **air is a fluid) caused by the weight of air** above the measurement point.

Atmospheric pressure is defined as the force per unit area exerted against a surface by the weight of the air above that surface. In the diagram below, the pressure at point "X" increases as the weight of the air above it increases. The same can be said about decreasing pressure, where the pressure at point "X" decreases if the weight of the air above it also decreases.



Several experiments can show that the atmosphere exerts a pressure and that this pressure is large. You can do the following two experiments at home.



EXPERIMENT 6.3 – To demonstrate atmospheric pressure

1. **Fill a cup to the brim with water**
2. **Slide a flat piece of cardboard over the top**
3. **While holding the cardboard in place, turn upside down the cup**
4. **Remove your hand from the cardboard.**

(Remember to do this experiment over the sink!)

Why does the water not fall out?

The explanation is that the atmospheric pressure pushing upwards on the cardboard is grater than the downwards pressure exerted by the water. So the cardboard stays in place.

EXPERIMENT 6.4 – To demonstrate atmospheric pressure

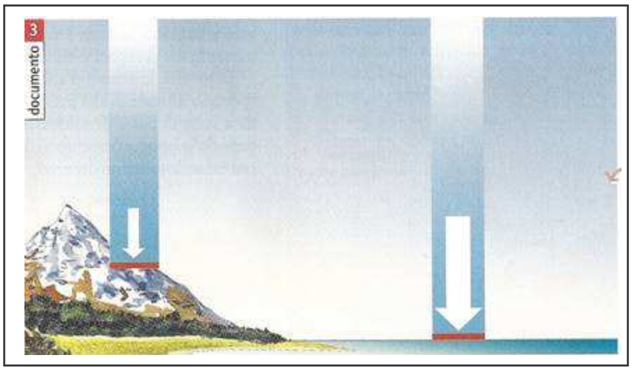
1. **Boil some water in a can o a tin for a minute or two (until all the air has been driven out by the steam).**
2. **Remove the source of heat and immediately screw on the cap on the can.**
3. **Cool the can under water**

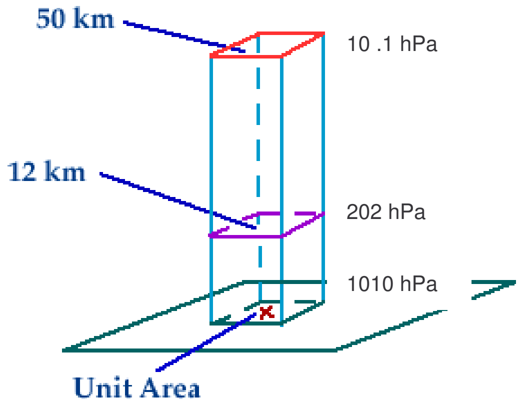
The can squash quite dramatically.
This happens because the steam condenses and leaves the vacuum inside the can. The atmospheric pressure which is pressing on the outside of the can squashes the can quite dramatically.

■ PRESSURE AND ALTITUDE → pressure decreases with increasing altitude

The atmosphere is like an invisible ocean of air and we are living at the bottom of it. It extends upwards for many kilometres. As the **height** of the atmosphere is greatest at sea level, so **the pressure of the atmosphere is greatest at the sea level.** As you **ascend**, there is less air pressing down and the **pressure falls** (**remember Stevin's law**, even if, unlike the pressure caused by the depth of water, there is no simple relationship between atmospheric pressure and height)**.**

At sea level, the pressure of the atmosphere is about 1010 hPa (hectopascals) or, that is the same, **101 kPa.**





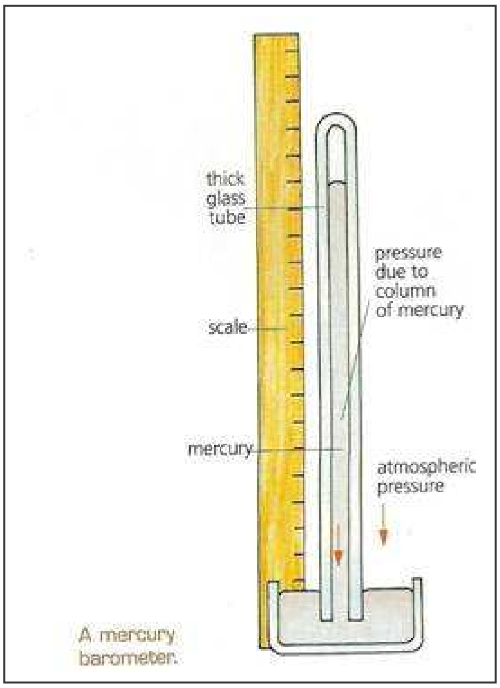
Atmospheric pressure decreases with height, dropping by 50% at an altitude of about 5.6 km. Equivalently, about 50% of the total atmospheric mass is within the lowest 5.6 km. This pressure drop is approximately exponential, so that pressure decreases by approximately half every 5.6 km. However, **because of changes in temperature throughout the atmospheric column, as well as the fact that the force of gravity begins to decrease at great altitudes, a single equation does not model atmospheric pressure through all altitudes**.

- 50% of the atmosphere by mass is below an altitude of 5.6 km.
- 90% of the atmosphere by mass is below an altitude of 16 km. The common altitude of commercial airliners is about 10 km.
- 99.99997% of the atmosphere by mass is below 100 km.

■ **PRESSURE AND TEMPERATURE → pressure decreases with increasing temperature**

Therefore, **where the temperatures are colder**, **a given pressure surface will have a lower height than if the same pressure surface was located in warmer air**.

6.2 – MEASURING ATMOSPHERIC PRESSURE: THE BAROMETER



A **barometer** is an instrument that measures air pressure. It was invented by **Evangelista Torricelli** (1608-1647) a famous Italian scientist. The simplest type is the mercury barometer. The pressure caused by the column of mercury (according to the Stevin's law) inside a glass tube is balanced by the atmosphere outside the glass tube. When the atmosphere pressure rises, the mercury is pushed higher in the tube, and so the height of the mercury in the tube is a measure of the atmospheric pressure.

Normal atmospheric pressure is about the same exerted by a column of mercury 760 mm high. That's why we can also say that **the atmospheric pressure (at the sea level) is 760 mmHg** and that **760 mmHg is equal to 101 kilopascals (or 1010 hectopascals)or 1 atmosphere: 760 mmHg = 1 atm =101 kPa.**

GLOSSARY

NOUNS, ADJECTIVES, ADVERBS

push	spinta, urto, impulso
skier	sciatore
sinking	affondamento
caterpillar track	cingolato
stake	palo, piolo
sharp	affilato
wheel	ruota
hammer	martello
needle	ago
dull	spuntato, smussato
blade	lama
car lift	ponte elevatore
lift truck	carrello elevatore
brake	freno
office safe	cassaforte
marble	marmo
braking gear	congegno dei freni
downwards	in giù, verso il basso
loss	perdita, spreco
rate	velocità
hole	buco, foro
hydraulic press	torchio idraulico
diver	tuffatore
eardrum	timpano
dam	diga
the bends	embolia gassosa
depth	profondità
vessel	vaso, recipiente
on the bottom	sul fondo
at the top	in alto
buoyant force	spinta idrostatica
upwards	verso l’alto
average	media
vacuum	vuoto (sost)
empty	vuoto (agg)
submerged	immerso
percentage	percentuale
tip of the iceberg	punta dell’iceberg
cork	tappo di sughero
ice cube	cubetto di ghiaccio
ice floe	blocco di ghiaccio
sink	lavandino della cucina
basin	vasca
chin	mento
independently of	indipendentemente da
brim	orlo, bordo
cardboard	cartone
can	recipiente metallico
tin	lattina
throughout	attraverso
millimetres of mercury (mmHg)	millimetri di mercurio

VERBS

to push	schiacciare, premere, spingere
to wonder	meravigliarsi, stupirsi, chiedersi, essere sorpreso
to sink	affondare
to peel	sbucciare
to lift	sollevare
to brake /to put on the brakes	frenare/azionare I freni
to press	premere, comprimere,calcare
to press down	schiacciare, comprimere, abbassare esercitando una pressione
to be made of /to be composed of	consistere, essere fatto di
to flow	fluire
to spray	spruzzare
to punch	forare, fare un buco con un punzone
to fill	riempire
to dive	tuffarsi
to take a dive	fare un tuffo
to dive into the heart of matter	entrare nel vivo della questione
to squeeze	comprimere
to withstand	contrastare
to bear in mind	tener presente
to spurt out	sprizzare, zampillare
to float	galleggiare
to enable	permettere
to write down	annotare
to displace	spostare
to surround	circondare
to re-emerge	riemergere
to rise up	sollevarsi
to equal	uguagliare
to meet with	imbattersi
to exhaust	esaurire, consumare
to cut by half	dimezzare
to double	raddoppiare
to hang	appendere
to breath out	espirare
to turn upside down	capovolgere

MATHEMATIC GLOSSARY

scalar (quantity)	grandezza scalare
vector (quantity)	grandezza vettoriale
directly proportional	direttamente proporzionale
inversely proportional	inversamente proporzionale
direct ratio	proporzionalità diretta
inverse proportion	proporzionalità in versa
= equals, is equal to	uguale
< less than	minore di
< greater than	maggiore di
***a + b* (a plus b)**	*a* più *b*
***a – b* (a minus b)**	*a* meno *b*
***a : b* (a divided by b)**	*a* diviso *b*
***a · b* (a times b)**	a per b
to add	addizionare
to subtract from	sottrarre
to multiply by	moltiplicare per
to divide by	dividere per
power	potenza
base	base
exponent	esponente
to raise	elevare
***a* squared**	*a* al quadrato
***a* cubed**	*a* al cubo
ten* to the *nth power	10 elevato alla n, 10^n
***a/b* a over b**	*a* su *b*
½one half	un mezzo